Advanced Topics on Weighted Tree Automata

Exercise 1 (Proving Lemma 4)
Lemma 4 (Stüber, Vogler, and Fülöp, 2009) essentially states that, given a multioperator monoid $\mathcal{A}$ and a ranked alphabet $\Sigma$, the weighted tree language $\lbrack M \rbrack$ recognized by a weighted tree automaton $M$ over $\mathcal{A}$ and $\Sigma$ can also be expressed by the composition $\lbrack \rho \rbrack; \chi_{\lbrack G,H \rbrack}; \lbrack M_{\text{hom}} \rbrack$, where

- $\rho : T_\Sigma \rightarrow \mathcal{P}_{\text{fin}}(T_\Delta)$ is a non-overlapping relabeling,
- $(G, H)$ is a local tree grammar over the alphabet $\Delta$, and $\chi_{\lbrack G,H \rbrack} : T_\Delta \rightarrow \mathcal{P}_{\text{fin}}(T_\Delta)$ is the partial identity of its generated local tree language,
- $M_{\text{hom}} = (Q_{\text{hom}}, \delta_{\text{hom}}, F_{\text{hom}})$ is a hom-wmta over $\mathcal{A}$ and $\Delta$, i.e., $|Q_{\text{hom}}| = |F_{\text{hom}}| = 1$.

The construction of $\rho$, $(G, H)$ and $M_{\text{hom}}$ has already been detailed in the lecture. Now complete the proof to assure yourself that these do indeed constitute a decomposition of $M$, i.e. that

$$\lbrack M \rbrack = \lbrack \rho \rbrack; \chi_{\lbrack G,H \rbrack}; \lbrack M_{\text{hom}} \rbrack.$$

Exercise 2 (FTA; WMTA($\mathcal{A}$) $\subseteq$ WMTA($\mathcal{A}$))
Presuming a ranked alphabet $\Sigma$, as well as an absorptive multioperator monoid $\mathcal{A}$, prove that, given an fta $B$ and a wmta $M$ over $\mathcal{A}$ and $\Sigma$, one can construct another wmta $M'$ over $\mathcal{A}$ and $\Sigma$ such that

$$\lbrack M' \rbrack = \chi_{\lbrack \mathcal{A} \rbrack}; \lbrack M \rbrack.$$

References